Övning 2

29 jan

J.11

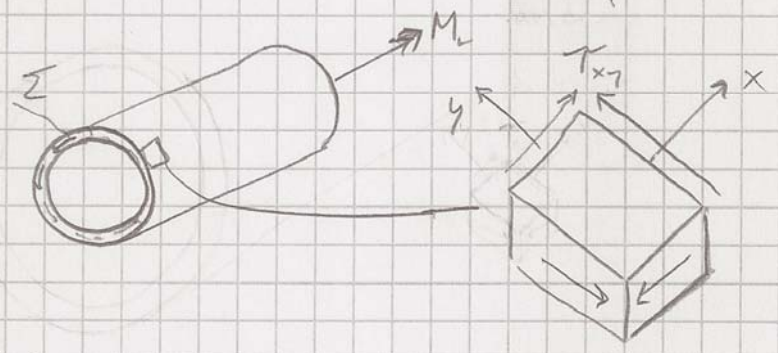
Tunnväggigt rör (inte varierande spänningar i tvärsnittet)

Söker Mohrs spänningscirklar.

Huvudspänningar

Effektivspänningar enl. Tresca och von Mises.

Lösning



Studerar ett litet element

$\sigma_x = \sigma_y = \sigma_z = 0$

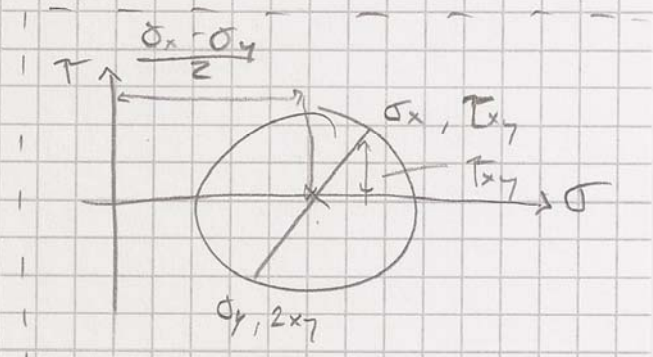
$\tau_{xy} = \tau_{yz} = 0$

Jämvikt

$\nearrow$: $M_v - \tau \underbrace{2\pi a \cdot h}_{\text{area}} \cdot a = 0$ ← hävarm

periferiell kraft

$$\tau_{xy} = -\tau = -\frac{M_v}{2\pi a^2 h}$$

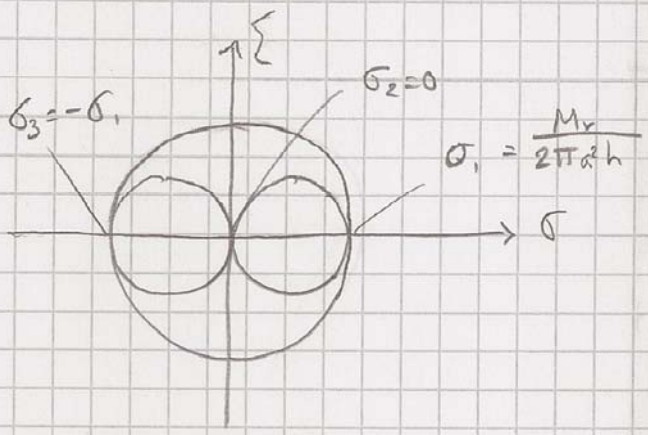


Ekv 9-84 sid 175 ger

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\left.\begin{aligned} \sigma_1 &= -\tau_{xy} = \frac{M_v}{2\pi a^2 h} \\ \sigma_2 &= 0 \\ \sigma_3 &= \tau_{xy} = -\frac{M_v}{2\pi a^2 h} \end{aligned}\right\} \quad \sigma_1 > \sigma_2 > \sigma_3$$

Mohrs spänningscirklar



$$\Rightarrow \tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{M_v}{2\pi a^2 h}$$

Tresca 1b sid 238 eku 12-14

$$\sigma_e^T = 2\tau|_{max} = \max\left(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\right) = \frac{M_v}{\pi a^2 h}$$

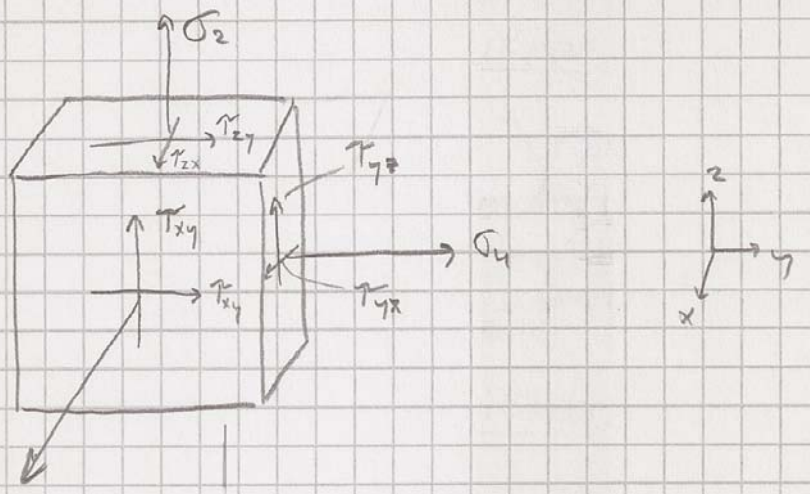
Von Mises 1b sid 235 eku 12-7

$$\sigma_e^M = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3} = \sqrt{\sigma_1^2 + (-\sigma_1)^2 - \sigma_1(-\sigma_1)} =$$

$$= \sigma_1\sqrt{3} = \frac{\sqrt{3}}{2}\frac{M_v}{\pi a^2 h}$$

$$= 0{,}866\,\frac{M_v}{\pi a^2 h} = \frac{1 - 0{,}866}{1} = 0{,}134$$

7.19



$\sigma_x = 60$ MPa $\sigma_y = 40$ MPa $\sigma_z = 25$ MPa

$\tau_{xy} = \tau_{yx} = 0$; $\tau_{xz} = \tau_{zx} = 30$ MPa

$\tau_{yz} = \tau_{zy} = 20$ MPa

(9-37) och (9-38) ger: (sid 162 mycket viktig)

$$\det(S - \sigma I) = \det \begin{vmatrix} \sigma_x - \sigma & \tau_{xy} & \tau_{zx} \\ \tau_{xy} & \sigma_y - \sigma & \tau_{yz} \\ \tau_{zx} & \tau_{yz} & \sigma_z - \sigma \end{vmatrix} = 0$$

(9-39) ger

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

(9-40) ger: Koefficienterna I_1, I_2, I_3

$$I_1 = \sigma_x + \sigma_y + \sigma_z = 60 + 40 + 25 = 125$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2 =$$

$$= 60 \cdot 40 + 40 \cdot 25 + 25 \cdot 60 - 0^2 - 30^2 - 20^2 = 3600$$

$$I_3 = \sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xy}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{xy}\tau_{yz}\tau_{zx} =$$

$$= 60 \cdot 40 \cdot 25 - 60 \cdot 20^2 - 40 \cdot 30^2 - 25 \cdot 0^2 + 2 \cdot 0 \cdot 30 \cdot 20 = 0$$

$$\sigma^3 - 125\sigma^2 + 3600\sigma = 0$$

En huvudspänning känd = 0

$\therefore (\sigma - 0)(\sigma^2 + A\sigma + B) = 0$

$\sigma^3 + A\sigma^2 + B\sigma = 0$

$\therefore A = -125$ och $B = 3600$

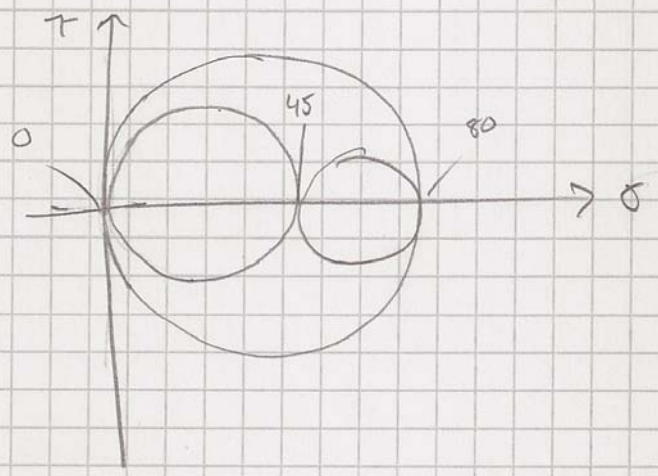
$\sigma^2 - 125\sigma + 3600 = 0$

$\sigma_{1,2} = \frac{125}{2} \pm \sqrt{\left(\frac{125}{2}\right)^2 - 3600} = 62{,}5 \pm 17{,}5$

$\therefore \sigma_1 = 80$ MPa

$\sigma_2 = 45$ MPa

$\sigma_3 = 0$



Effektivspänningarna

Tresca 1b sid 238 ekv (12-14)

$$\sigma_e^T = 2|\tau|_{max} = |\sigma_1 - \sigma_3| = |80-0| = 80 \text{ MPa}$$

von Mises 1b sid 235

$$\sigma_e^M = \sqrt{\frac{1}{2}(\sigma_1-\sigma_2)^2 + \frac{1}{2}(\sigma_2-\sigma_3)^2 + \frac{1}{2}(\sigma_3-\sigma_1)^2} =$$

$$= \sqrt{\frac{1}{2}(80-43)^2 + \frac{1}{2}(45-0)^2 + \frac{1}{2}(0-80)^2} = 69{,}5 \text{ MPa}$$