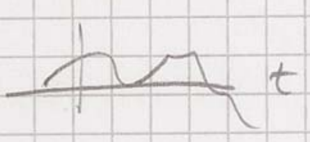
Övning 2 LV2 — 28 jan — Hana

z-transformen

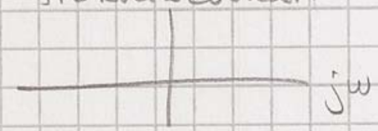
Fouriertransform $\quad X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

Laplacetransform $\quad X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st} \quad$ där $\quad s = \sigma + j\omega$

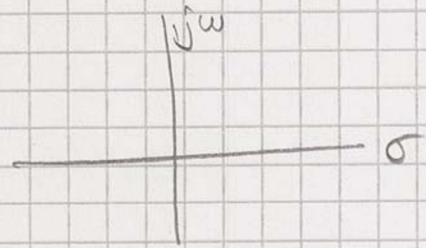
tidsdomän $\qquad\qquad\qquad$ frekvensdomän



$\xleftrightarrow{FT}$



tidsdomän $\quad \xleftrightarrow{LT}$



Diskret FT : $\quad X[\Omega] = \sum_{-\infty}^{\infty} x[n]\, e^{-jn\Omega}$

Via att sätta $\quad z = \Sigma + j\Omega \quad$ får vi :

z-transform $\quad F(z) = \sum_{-\infty}^{\infty} x[n]\, z^{-n}$

9.1 Express the unilateral z-transforms of the following function as rational functions. Tables may be used.

a)

$$x[n] = 0{,}3^n$$

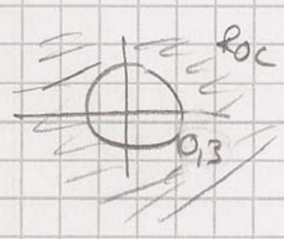
$$\mathcal{Z}\{0{,}3^n u[n]\} = \sum_{n=0}^{\infty} 0{,}3^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{0{,}3}{z}\right)^n =$$

a

Från tabell:

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \quad |a|<1 \Rightarrow \frac{1}{1-\frac{0{,}3}{z}} = \frac{z}{z-0{,}3}$$

$$\left|\frac{0{,}3}{z}\right| < 1 \quad \overset{ROC}{\Longrightarrow} \quad |z| > 0{,}3$$



c)

$$x[n] = 3e^{-0{,}7n}$$

$$\mathcal{Z}\{3e^{-0{,}7n} u[n]\} = \sum_{n=0}^{\infty} 3e^{-0{,}7n} \cdot z^{-n} = 3\sum \left(\frac{e^{-0{,}7}}{z}\right)^n$$

$$= \left\{\begin{matrix} \text{App} \\ \text{C} \end{matrix}\right\} = 3\,\frac{1}{1-\frac{e^{-0{,}7}}{z}} = \frac{3z}{z-e^{-0{,}7}}$$

ROC:

$$\left|\frac{e^{-0{,}7}}{z} < 1\right| \Rightarrow z > e^{-0{,}7}$$

e)

$$5\cos(3n) \Rightarrow \left\{ \begin{array}{l} \text{euler} \\ \cos(\phi) = \frac{e^{j\phi} + e^{-j\phi}}{2} \end{array} \right\}$$

$$\mathcal{Z}\{x[n]\} = \sum_{n=0}^{\infty} 5 \, \frac{e^{j3n} + e^{-j3n}}{2} z^{-n} =$$

$$= \frac{5}{2}\left(\sum_{n=0}^{\infty} \left(\frac{e^{3j}}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{e^{-3j}}{z}\right)^n \right) = \left\{ \begin{array}{c} \text{App} \\ \text{C} \end{array} \right\} =$$

$$= \frac{5}{2}\left(\frac{1}{1 - \frac{e^{3j}}{z}} + \frac{1}{1 - \frac{e^{-3j}}{z}} \right) = \frac{5}{2}\left(\frac{z}{z - e^{3j}} + \frac{z}{z - e^{-3j}} \right)$$

$$= \frac{5}{2}\left(\frac{z^2 - ze^{-j3} + z^2 - ze^{3j}}{(z - e^{3j})(z - e^{-3j})} \right) = \frac{5}{2}\left(\frac{2z^2 - z(e^{3j} + e^{-3j})}{z^2 - z(e^{3j} + e^{-3j}) + 1} \right)$$

$$\left\{ \frac{e^{3j} + e^{-3j}}{2} \right\} = 5 \, \frac{z(z - \cos 3)}{z^2 - 2z\cos 3 + 1}$$

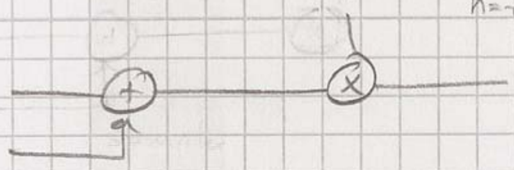
ROC: $|z| > 1$

9.2

The signals given are sampled every 0,05 s, beginning at $t=0$. Find the unilateral z-transforms of the sampled functions.

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

b/ $2e^{-2t} + 2e^{t}$



$T = 0{,}05 \text{ s} \implies t = nT = 0{,}05n$

$$f[n] = 2 \cdot e^{-2 \cdot 0{,}05n} + 2e^{0{,}05n} = 2\left(e^{-0{,}1n} + e^{0{,}05n}\right) =$$

$$\mathcal{Z}\{f[n]\} = 2\left(\frac{z}{z - e^{-0{,}1}} + \frac{z}{z - e^{0{,}05}}\right)$$

d/ $5e^{-0{,}5jt}$

$$f[n] = 5 \cdot e^{-0{,}5j \cdot 0{,}05n} = 5e^{-j \cdot 0{,}025n}$$

$$\mathcal{Z}\{f[n]\} = \frac{5z}{z - e^{-j \cdot 0{,}025}}$$

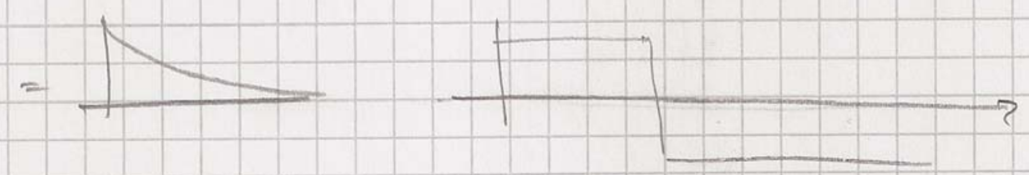
HV 7.17 Determine the z-transform for the following time signals

d) $x[n] = \left(\frac{1}{4}\right)^n \left(u[n] - u[n-5]\right)$

Alternativ 1

Def. of z-transform

$$X(z) = \sum_{-\infty}^{\infty} \left(\frac{1}{4}\right)^n \left(u[n] - u[n-5]\right) z^{-n} =$$



$$= \sum_{n=0}^{4} \left(\frac{1}{4}\right)^n z^{-n} = \sum_{0}^{4} \left(\frac{1}{4z}\right)^n = \left\{ \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a} \right\} =$$

$$= \frac{1 - \left(\frac{1}{4z}\right)^5}{1 - \frac{1}{4z}}$$

Alternativ 2

real-shifting property

$$\mathcal{Z}\left[f[n-n_0]\, u[n-n_0]\right] = \sum_{n=n_0}^{\infty} f[n-n_0]\, z^{-n} = z^{-n_0} F(z)$$

$$x[n] = \left(\frac{1}{4}\right)^n u[n] - \left(\frac{1}{4}\right)^{n} u[n-5] \rightarrow n = n-5+5$$

$$= \left(\frac{1}{4}\right)^n u[n] - \left(\frac{1}{4}\right)^5 \cdot \left(\frac{1}{4}\right)^{n-5} u[n-5] =$$

$$= \frac{z}{z-\frac{1}{4}} - \frac{z}{z-\frac{1}{4}} \cdot \left(\frac{1}{4}\right)^5 \cdot z^{-5} = \frac{1 - \left(\frac{1}{4z}\right)^5}{z - \frac{1}{4z}}$$

Alternativ 3

$$\sum_{n=0}^{4} \left(\frac{1}{4}\right)^n z^{-n} = \sum_{n=0}^{4} \left(\frac{1}{4z}\right)^n = \frac{1}{4z^0} + \frac{1}{4z^1} + \frac{1}{4z^2} +$$

$$+ \frac{1}{4z^3} + \frac{1}{4z^4}.$$

Gör till bättre form :(

9.4

a) Use the z-transform to evaluate the following Series

(i) $x = \sum_{n=0}^{\infty} (0,3)^n = ?$

$$\mathcal{Z}\{0,3^n\} = \sum_{n=0}^{\infty} 0,3^n z^{-n} = \frac{z}{z-0,3} = \{z=1\} = \underline{\underline{\frac{10}{7}}}$$

(ii)

$$x = \sum_{n=5}^{\infty} (0,3)^n \longleftrightarrow ?$$

$$\mathcal{Z}\{0,3^{n-5+5} u[n-5]\} = \sum_{n=0}^{\infty} 0,3^{n-5} z^{-n} =$$

$$= 0,3^5 z^{-5} \cdot \frac{z}{z-0,3} = \{z=1\} = \underline{\underline{0,3^5 \cdot \frac{1}{0,7}}}$$

Det viktiga i de här fallen är transformen av tidsförskjutningen.