S3 övning 1 9/2

HV 7.27

Determine the impulse corresponding to the following functions. The systems are causal.

c/ $X(z) = \dfrac{4z}{z^2 - \frac{1}{2}z + \frac{1}{16}}$, $x[n] = ?$

Strategy:

Skriv om $X(z)$ något och plocka från tabell

Utförande:

$$X(z) = \frac{4z \cdot \frac{1}{4} \cdot 4}{(z - \frac{1}{4})^2} \qquad \Bigg\| \qquad n a^n \longleftrightarrow \frac{az}{(z-a)^2}$$

$\longrightarrow a = \frac{1}{4}$

$$x[n] = 16\, n \left(\tfrac{1}{4}\right)^n u[n] \qquad \text{ROC}: |z| > \tfrac{1}{4}$$

HV 7.29

A casual system has input $x[n]$ and output $y[n]$. Determine the impulse response to the system.

b) $x[n] = (-3)^n u[n]$

$y[n] = 4 \cdot (2)^n u[n] - \left(\frac{1}{2}\right)^n u[n]$

$h[n] = ?$

Strategi:

- z-transformera
- $Y(z) = X(z) H(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)}$
- invers z-transform tillbaka för att få $h[n]$

Utförande

$$x[n] \xleftrightarrow{ZT} \frac{z}{z-(-3)} = \frac{z}{z+3} = X(z)$$

$$y[n] \xleftrightarrow{ZT} 4 \cdot \frac{z}{z-2} - \frac{z}{z-\frac{1}{2}} = Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{4z(z+3)}{(z-2)z} - \frac{\cancel{z}(z+3)}{(z-\frac{1}{2})\cancel{z}} =$$

$$= \frac{4(z+3)}{z-2} - \frac{z+3}{z-\frac{1}{2}} = 4H_1(z) - H_2(z)$$

$$\frac{H_1(z)}{z} = \frac{z+3}{z(z-2)} = \frac{k_1}{z} + \frac{k_2}{z-2} \quad \Bigg\| \quad k_1 = -\frac{3}{2}, \quad k_2 = \frac{5}{2}$$

$$H_1(z) = z\left(\frac{-\frac{3}{2}}{z} + \frac{\frac{5}{2}}{z-2}\right) = \underline{\underline{-\frac{3}{2} + \frac{5}{2}\,\frac{z}{z-2}}}$$

$$\frac{H_2(z)}{z} = \frac{z+3}{z\left(z-\frac{1}{2}\right)} = \frac{k_1}{z} + \frac{k_2}{z-\frac{1}{2}} \quad \Bigg\| \quad \begin{aligned} k_1 &= -6 \\ k_2 &= 7 \end{aligned}$$

$$H_2(z) = z\left(\frac{-6}{z} + \frac{7}{z-\frac{1}{2}}\right) = -6 + \frac{7z}{z-\frac{1}{2}}$$

$$H(z) = 4H_1(z) - H_2(z) = 4\left(\frac{5}{2}\,\frac{z}{z-2} - \frac{3}{2}\right) - \left(-6 + \frac{7z}{z-\frac{1}{2}}\right)$$

$$\underline{\underline{h[n] = \left(10(2)^n - 7\left(\tfrac{1}{2}\right)^n\right)u[n]}}$$

Alternativt räknesätt:

$$X(z) = \frac{z}{z+3} = \frac{1}{1+3z^{-1}}$$

$$Y(z) = \frac{4z}{z-2} - \frac{z}{z-\frac{1}{2}} = \frac{4}{1-2z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = 4\left(\frac{1+3z^{-1}}{1-2z^{-1}}\right) - \frac{1+3z^{-1}}{1-\frac{1}{2}z^{-1}} =$$

$$= 4\left(\frac{1}{1-2z^{-1}} + \frac{3z^{-1}}{1-2z^{-1}}\right) - \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{3z^{-1}}{1-\frac{1}{2}z^{-1}} =$$

$$= 4\left(\frac{z}{z-2} + \frac{3z\cdot z^{-1}}{z-2}\right) - \left(\frac{z}{z-\frac{1}{2}} + \frac{3z^{-1}\cdot z}{z-\frac{1}{2}}\right)$$

$$h[n] = 4\left(2^n u[n] + 3\left(2^{n-1}u[n-1]\right)\right) - \left(\left(\tfrac{1}{2}\right)^n u[n] + 3\left(\left(\tfrac{1}{2}\right)^{n-1}u[n-1]\right)\right)$$

Kom ihåg:

$\delta[n-n_0] \longleftrightarrow z^{-n_0}$

Alltså z^{-1} ger tidsförskjutning i tidsdomänet

Slutsats:

Olika svar, men båda är korrekta

HV 7.30

A system has impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$.
Determine the input $x[n]$ to the system if the output is given by:

b/ $y[n] = \frac{1}{3} u[n] + \frac{2}{3}\left(-\frac{1}{2}\right)^n u[n]$

Strategi:

- Z-transformera $h[n]$ och $y[n]$
- Nyttja att $Y(z) = X(z) \cdot H(z) \Rightarrow X(z) = \frac{Y(z)}{H(z)}$
- Transformera tillbaka

Utförande:

$$Y(z) = \frac{1}{3} \frac{z}{z-1} + \frac{2}{3} \frac{z}{z+\frac{1}{2}}$$

$$H(z) = \frac{z}{z-\frac{1}{2}}$$

$$X(z) = \frac{Y(z)}{H(z)} = \frac{1}{3} \frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}} + \frac{2}{3} \frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{2}} =$$

$$\frac{1}{3} \frac{1-z^{-1}+z^{-1}-\frac{1}{2}z^{-1}}{1-z^{-1}} + \frac{2}{3} \frac{1+\frac{1}{2}z^{-1}-\frac{1}{2}z^{-1}-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}} =$$

$$= \frac{1}{3}\left(1 + \frac{\frac{1}{2}z^{-1}}{1-z^{-1}}\right) + \frac{2}{3}\left(1 - \frac{z^{-1}}{1+\frac{1}{2}z^{-1}}\right) =$$

$$= 1 + \frac{1}{6} \frac{z^{-1}}{1-z^{-1}} - \frac{2}{3} \frac{z^{-1}}{1+\frac{1}{2}z^{-1}}$$

$$x[n] = \delta[n] + \frac{1}{6} u[n-1] - \frac{2}{3}\left(-\frac{1}{2}\right)^{n-1} u[n-1]$$

7.32

Determine the transfer function and difference equation representation of the systems with the following impulse responses.

b) $h[n] = \left(\frac{1}{3}\right)^{n} u[n] + \underbrace{\left(\frac{1}{2}\right)^{n-2} u[n-1]}$

$$f[n-n_0]\,u[n-n_0] = z^{-n_0} F(z)$$

$$h[n] = \left(\frac{1}{3}\right)^{n} u[n] + 2 \cdot \left(\frac{1}{2}\right)^{n-1} u[n-1]$$

$$H(z) = \frac{z}{z-\frac{1}{3}} + z^{-1} \cdot \frac{2z}{z-\frac{1}{2}} = \frac{z\left(z-\frac{1}{2}\right) + 2\left(z-\frac{1}{3}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{2}\right)}$$

$$H(z) = \frac{z^2 + \frac{3}{2}z - \frac{2}{3}}{z^2 - \frac{5}{6}z + \frac{1}{6}} = \frac{1 + \frac{3}{2}z^{-1} - \frac{2}{3}z^{-2}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} = \frac{Y(z)}{X(z)}$$

$$Y(z) = \left(1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right) = X(z)\left(1 + \frac{3}{2}z^{-1} - \frac{2}{3}z^{-2}\right)$$

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n] + \frac{3}{2}x[n-1] - \frac{2}{3}x[n-2]$$

Strategi för denna typ av uppgift är alltså:

- z-transformera
- Kom ihåg hur en differensekvation representeras
- Skriv om till att passa detta
- Transformera tillbaka hela uttrycket.