SSY130 - Applied Signal Processing

Hand-In Problem 3 - Target Tracking Using the Kalman Filter

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1 Task 1 (theory)

$$\dot{x}(t)|_{t=Kt} \approx \frac{x(kT+T)-x(kT)}{T}$$

Rewritten as:

$$x(kT+T) = T\dot{x}(Kt) + x(kT)$$

States expressed in discrete time:

$$x(k+1) = x(k) + T\dot{x}$$
$$\dot{x}(k+1) = \dot{x}(k) + w_x$$
$$y(k+1) = y(k) + T\dot{y}(k)$$
$$\dot{y}(k+1) = \dot{y}(k) + w_y(k)$$

States expressed in s(k):

$$s_1(k+1) = s_1(k) + Ts_2$$
$$s_2(k+1) = s_2(k) + w_x$$
$$s_3(k+1) = s_3(k) + TS_4\,(k)$$
$$s_4(k+1) = s_4(k) + w_y(k)$$

Output expressed in discrete time:

$$z_1(k) = s_1(k) + v_x(k)$$
$$z_2 = s_3(k) + v_y(k)$$

Discrete state space model:

$$s(k+1) = \underbrace{\begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A} s(k) + \begin{bmatrix} 0 \\ w_x \\ 0 \\ w_y \end{bmatrix}$$

$$z(k+1) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{C} s(k) + \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

2 Task 2 (Matlab)

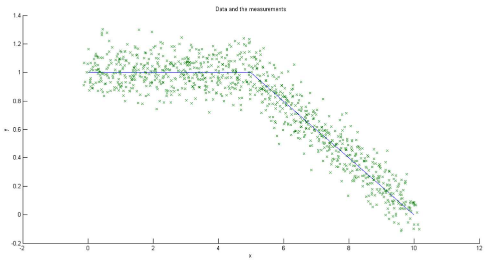

Figure 1: Original data and measurement with noise plotted together with the measured signal. The noise level is Gaussian with mean 0 and standard distribution 0.1.

3 Task 3 (Matlab)

See uploaded code in course portal.

4 Task 4 (Matlab)

4.1 Values for T, Q and R

For every sample the x position is increased with 0.01 and the y value is changing between 0 and -0.002. This means that $\dot{x} = 0.1$ and $\max(|\dot{y}|) = 0.002$. The step size T has to be small enough to catch these derivates. $T = 0.01$ is used since a smaller T caused the calculated velocity to behave strange.

Q is the covariance of the w(k) process noise. R is the covariance of the observation noise v(k). The off diagonal elements in Q indicate the correlation between the state variables. No correlation between the disturbances in the state variables is assumed. The measurement disturbances are also assumed to be uncorrelated (since they are random). These two assumptions will result in diagonal matrices for both Q and R.

Also assumed is that the uncertainties is greater in the measurements (observation noise) compared to the uncertainties in the model (process noise). A starting point will therefore be to let the covariance matrix of the process noise, Q, be a 4x4 identity matrix. The covariance measurement noise matrix will be the identity matrix times a scalar with different values representing different ratios between Q and R. Greater values on the diagonal in the covariance matrix of the measurement noise will mean that the uncertainties in the measurement is assumed to be greater than the uncertainties in the model.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$$

$$r = \{1,10,100,1000\}$$

4.2 Matlab code

See uploaded code in course portal.

4.3 Plots

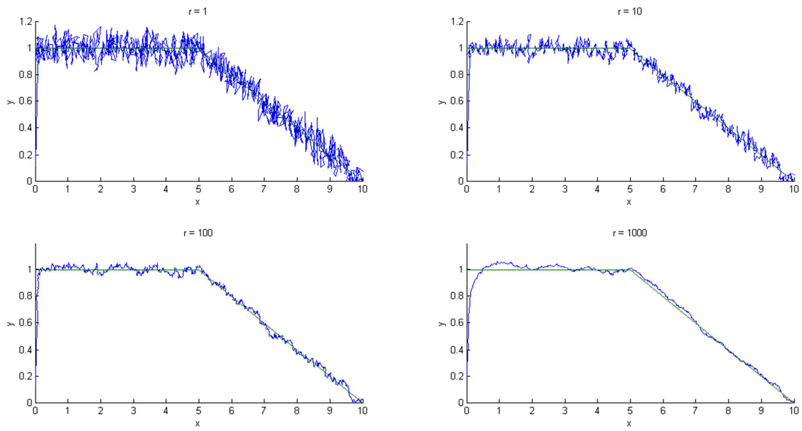

Figure 2: Kalman filter with different R applied to observed data (blue line) and the signal without noise (green line).

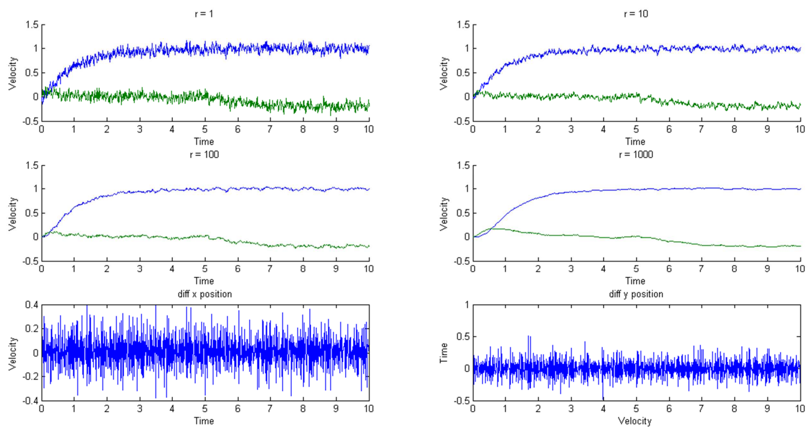

Figure 3: Kalman filter with different R applied to observed data. Velocity in x direction is the blue line. Velocity in y direction is the green line. The last two plots show the calculated velocity with the diff command on the measured data.